Introduction to Formal Concept Analysis

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Outline for the Tutorial:

- Presentation
- Short Break
- Practical Session

Outline for the Presentation:

- What is FCA ?
- Motivation
- What is a Formal Concept ?
- Formal Contexts
- Data Sets
- The Concept Lattice: Reading, drawing, interpretation
- Conceptual Scaling
- Summary

Formal Concept Analysis is ...

... is a mathematization of the philosophical understanding of concept

... a human-centered method to structure and analyze data

... a method to visualize data and its inherent structures, implications and dependencies

Motivation

... example from ToscanaJ

What is a concept ?

I

Let's examine an example, the concept “car”:

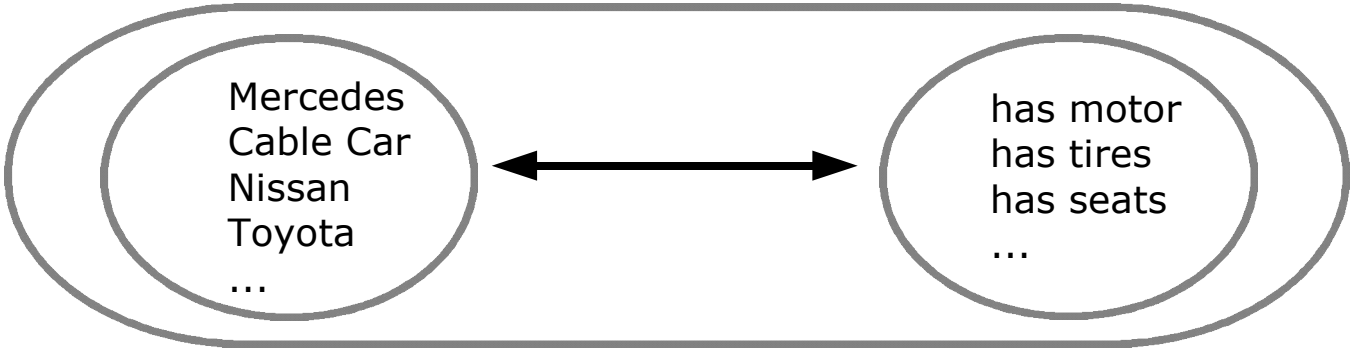
What drives us to call an object a “car” ?

- Every object having certain attributes is called “car”:
  - a car has tires
  - a car has a motor
  - a car has the purpose of transportation
  - a car has seats ... etc.
- All objects having these attributes are called “cars”:
  - Mercedes, Nissans, Toyotas are cars
  - San Francisco's Cable Car is called car ...

What is a concept ? II

This description of the concept “car” is based on sets of

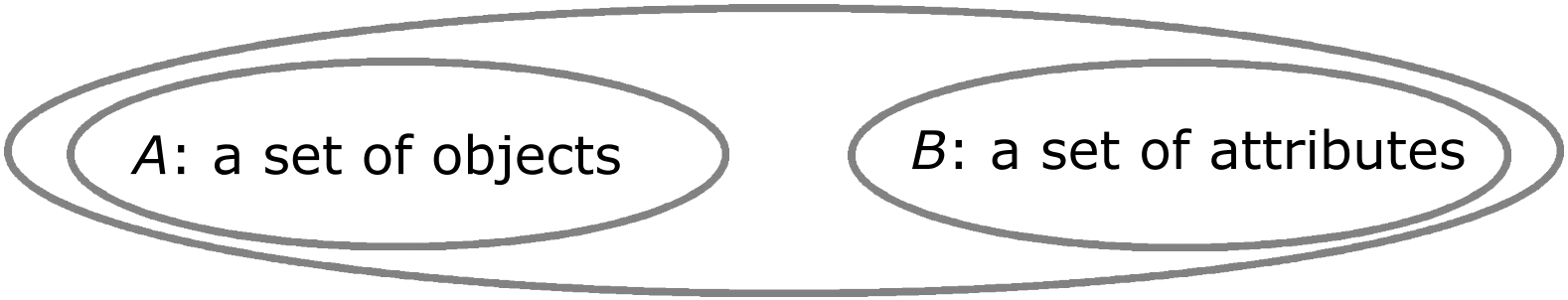
objects related to attributes:



→ Objects, attributes and a relation form a concept.

What is a concept ? III

So the concept is constituted by two parts ...

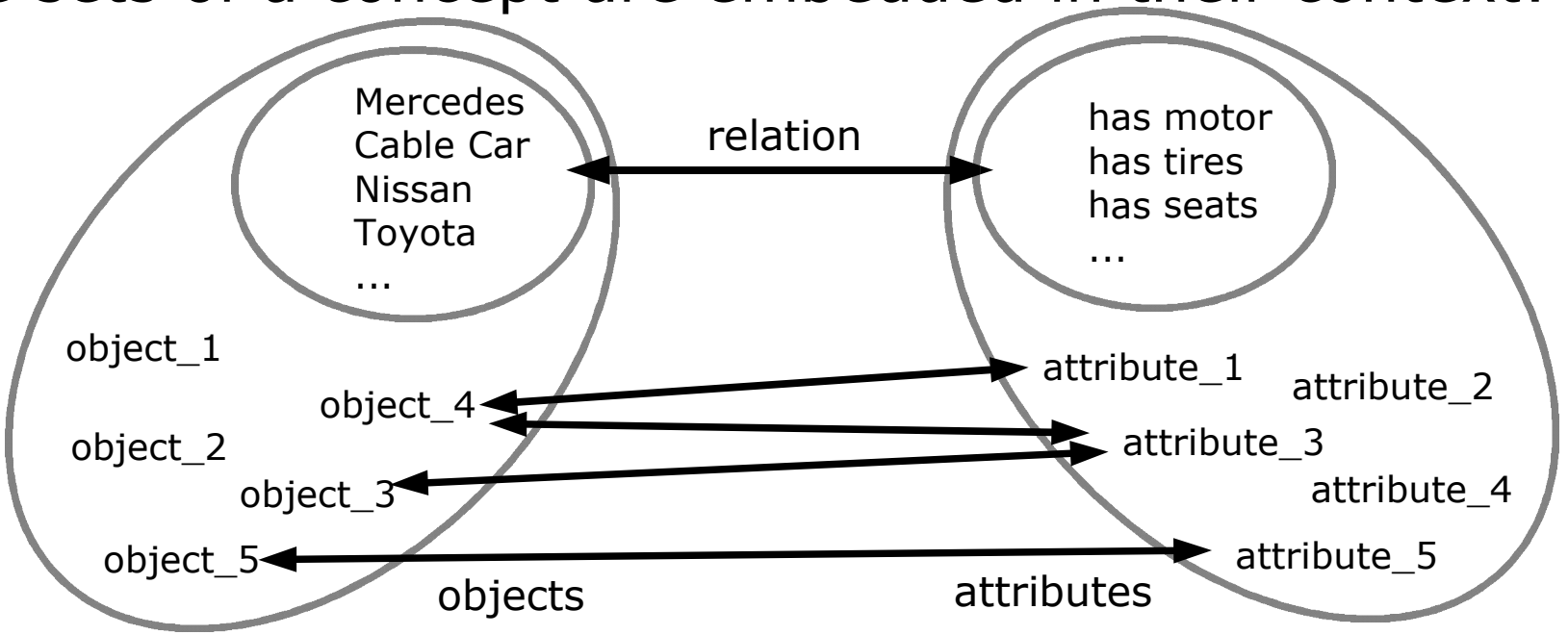


... having a certain relation:

- all objects belonging to this concept have all the attributes of *B*
- all attributes belonging to this concept are shared by all objects of *A*
- *A* is called the concept's *extension, B* is called the concept's *intension*

The Formal Context – Universe of Discourse I

The sets of a concept are embedded in their context:



These sets and their relations are the basis for all conclusions we make – any concept we derive, any implication we deduce is based on the context - it is our universe of discourse.

-> Changing the context will change the concepts and their structure

The Formal Context – Universe of Discourse II

M: a set of attributes

G: a set of objects

	small	medium	big	twolegs	fourlegs	feathers	hair	fly	hunt	run	swim	mane	hooves
dove	x	.	.	x	.	x	.	x	.	.	.	.	.
hen	x	.	.	x	.	x	.	.	.	.	.	.	.
duck	x	.	.	x	.	x	.	x	.	.	x	.	.
goose	x	.	.	x	.	x	.	x	.	.	x	.	.
owl	x	.	.	x	.	x	.	x	x	.	.	.	.
hawk	x	.	.	x	.	x	.	x	x	.	.	.	.
eagle	.	x	.	x	.	x	.	x	x	.	.	.	.
fox	.	x	.	.	x	.	x	.	x	x	.	.	.
dog	.	x	.	.	x	.	x	.	.	x	.	.	.
wolf	.	x	.	.	x	.	x	.	x	x	.	x	.
cat	x	.	.	.	x	.	x	.	x	x	.	.	.
tiger	.	.	x	.	x	.	x	.	x	x	.	.	.
lion	.	.	x	.	x	.	x	.	x	x	.	x	.
horse	.	.	x	.	x	.	x	.	.	x	.	x	x
zebra	.	.	x	.	x	.	x	.	.	x	.	x	x
cow	.	.	x	.	x	.	x	.	.	.	.	.	x

An incidence relation *I* between *G* and *M*

Such a group *(G,M,I)* of objects *G*, attributes *M* and a relation *I* is called a *formal context*.

There are many ways to transform more complex data into *formal contexts*.

The Formal Context – Universe of Discourse III

M: a set of attributes

G: a set of objects

	small	medium	big	twolegs	fourlegs	feathers	hair	fly	hunt	run	swim	mane	hooves
dove	x	.	.	x	.	x	.	x	.	.	.	.	.
hen	x	.	.	x	.	x	.	.	.	.	.	.	.
duck	x	.	.	x	.	x	.	x	.	.	x	.	.
goose	x	.	.	x	.	x	.	x	.	.	x	.	.
owl	x	.	.	x	.	x	.	x	x	.	.	.	.
hawk	x	.	.	x	.	x	.	x	x	.	.	.	.
eagle	.	x	.	x	.	x	.	x	x	.	.	.	.
fox	.	x	.	.	x	.	x	.	x	x	.	.	.
dog	.	x	.	.	x	.	x	.	.	x	.	.	.
wolf	.	x	.	.	x	.	x	.	x	x	.	x	.
cat	x	.	.	.	x	.	x	.	x	x	.	.	.
tiger	.	.	x	.	x	.	x	.	x	x	.	.	.
lion	.	.	x	.	x	.	x	.	x	x	.	x	.
horse	.	.	x	.	x	.	x	.	.	x	.	x	x
zebra	.	.	x	.	x	.	x	.	.	x	.	x	x
cow	.	.	x	.	x	.	x	.	.	.	.	.	x

Transposing the matrix, changing objects and attributes, creates the dual structure – the same diagram, but flipped top down.

The resulting implications and relations are the same.

Data Sets: Definition of Formal Concepts I

- For the mathematical definition of *formal concepts* we introduce the derivation operators " ' ".
  For a set of objects A, A' is defined as:
  A' = (all attributes in *M* shared by the objects of A)
  For a set of attributes B, B' is defined as:
  B' = (all objects in *G* that have all attributes of B).
- We are looking for pairs of sets (A,B) of objects and attributes that fulfill the conditions A' = B and B' = A and we call these pairs *formal concepts*.

Data Sets: Generating Formal Concepts II

Using the derivation operators we can derive *formal concepts* from our *formal context* with the following routine:

1) Pick a set of objects A.
2) Derive the attributes A'.
3) Derive (A')'.
4) (A'',A') is a *formal concept.*

A dual approach can be taken starting with an attribute.

Data Sets: Generating Formal Concepts III

	small	medium	big	twolegs	fourlegs	feathers	hair	fly	hunt	run	swim	mane	hooves
dove	x	.	.	x	.	x	.	x	.	.	.	.	.
hen	x	.	.	x	.	x	.	.	.	.	.	.	.
duck	x	.	.	x	.	x	.	x	.	.	x	.	.
goose	x	.	.	x	.	x	.	x	.	.	x	.	.
owl	x	.	.	x	.	x	.	x	x	.	.	.	.
hawk	x	.	.	x	.	x	.	x	x	.	.	.	.
eagle	.	x	.	x	.	x	.	x	x	.	.	.	.
fox	.	x	.	.	x	.	x	.	x	x	.	.	.
dog	.	x	.	.	x	.	x	.	.	x	.	.	.
wolf	.	x	.	.	x	.	x	.	x	x	.	x	.
cat	x	.	.	.	x	.	x	.	x	x	.	.	.
tiger	.	.	x	.	x	.	x	.	x	x	.	.	.
lion	.	.	x	.	x	.	x	.	x	x	.	x	.
horse	.	.	x	.	x	.	x	.	.	x	.	x	x
zebra	.	.	x	.	x	.	x	.	.	x	.	x	x
cow	.	.	x	.	x	.	x	.	.	.	.	.	x

1) Pick any set of objects A, e.g. A={duck}.

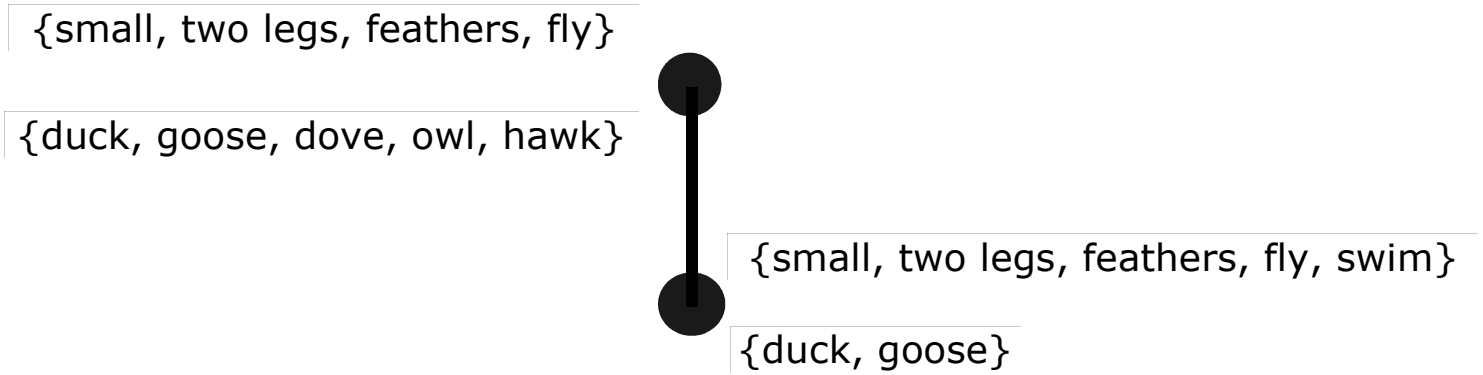
2) Derive the attributes A'={small, two legs, feathers, fly, swim}

3) Derive (A')'={small, two legs, feathers, fly, swim}'={duck, goose}

4) (A'',A')=({duck, goose},{small, two legs, feathers, fly, swim}) is a *formal concept.*

The *Concept Lattice* I

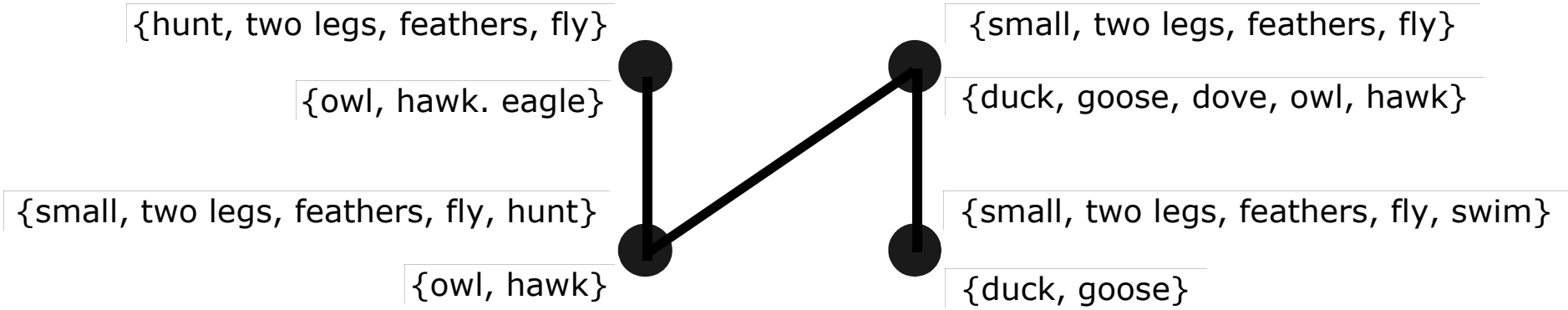
- The *formal concept* (A'',A')=({duck, goose},{small, two legs, feathers, fly, swim}) is represented in a line diagram as a node:



- Consider another *formal concept:* (B'',B')=({duck, goose, dove, owl, hawk},{small, two legs, feathers, fly}).
- The *formal concept* (A'',A') is called *subconcept* of (B'',B') and (B'',B') is called *superconcept* of (A'',A').
- (A'',A') is drawn below (B'',B') and connected with a line.

The *Concept Lattice* II

- By adding more *formal concepts* the diagram is extended step by step:
- ({owl, hawk},{feathers, two legs, small, fly, hunt})
- ({owl, hawk, eagle},{feathers, two legs, fly, hunt})
- ... plus the relations ...



- ... and so on ...
- Several methods exist to derive all *formal concepts*: Ganter's algorithm, cut over extents, Lindig's algorithm etc.

The *Concept Lattice* III

The *subconcept – superconcept* relation defines an order on the set $\underline{B}$ of all *formal concepts* of a *formal context*.

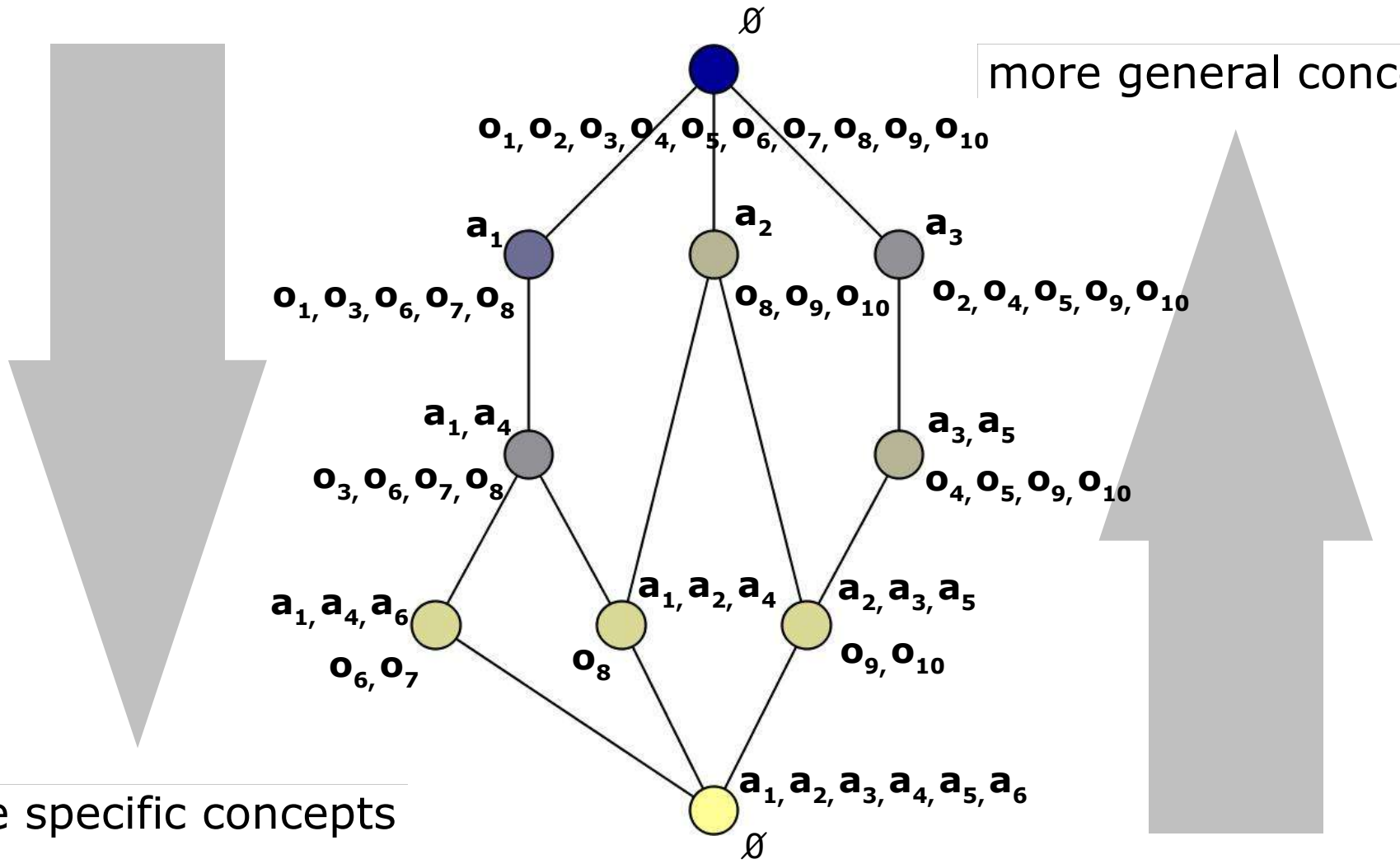
- For two concepts (A,B) and (C,D) this order is formalized as:
  $(A,B) \leqslant (C,D) :\Leftrightarrow A \subseteq C (\Leftrightarrow D \subseteq B)$
  (A,B) is smaller than (C,D) if A is subset of C (and D is subset of B)
- $(\underline{B},\leq)$ is an ordered set.

The set $\underline{B}$ of *formal concepts* has another property:

- for each set of *formal concepts* of a *formal context* there exists always a unique greatest *subconcept (meet)* and a unique smallest *superconcept (join)*.

The ordered set $(\underline{B},\leq)$ plus the last property forms a mathematical structure: the *concept lattice*.

The *Concept Lattice* IV

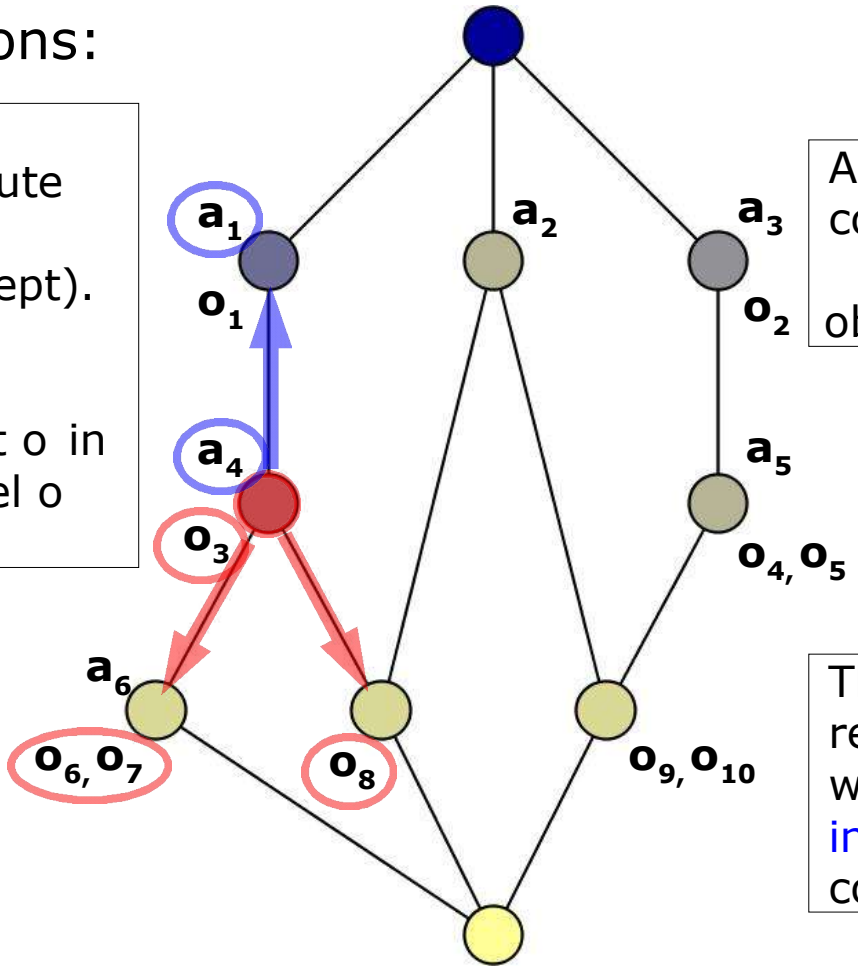


The *Concept Lattice* V

Labelling conventions:

The greatest concept having a certain attribute a in its intent gets the label a (attribute concept).

The smallest concept having a certain object o in its extent gets the label o (object concept).



Attributes above the concept -

objects below.

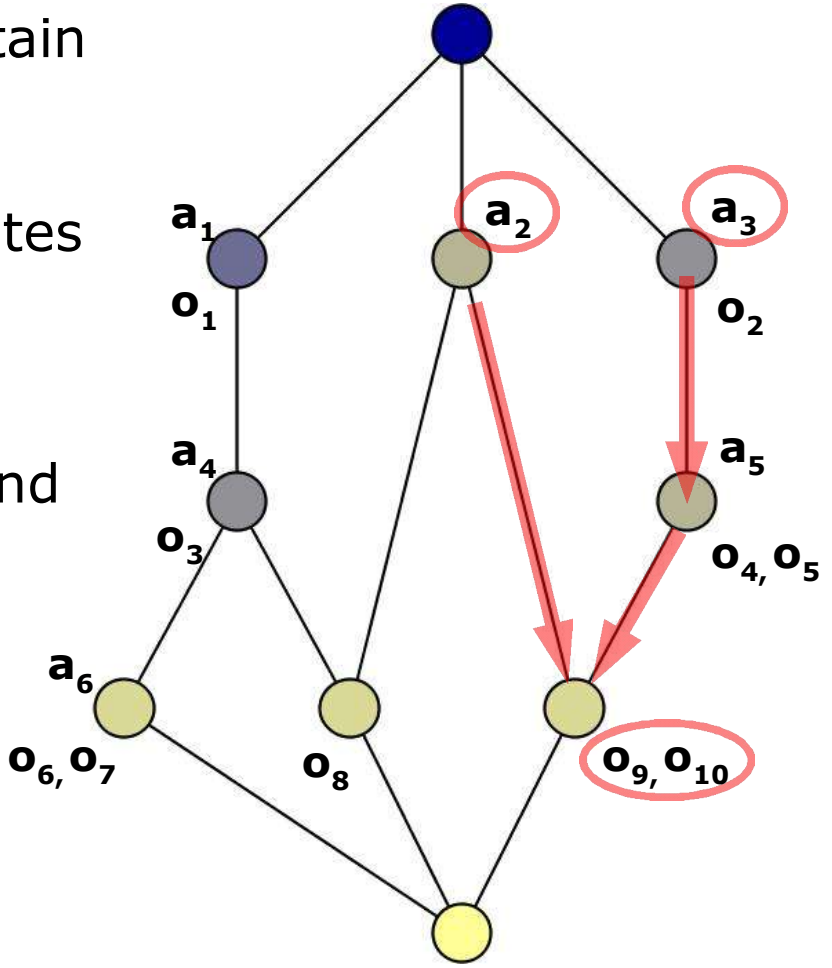
This allows the reconstruction of the whole extents and intents for every concept.

The *Concept Lattice:* Conjunction of concepts VI

Looking for objects with certain attributes:

Which object has the attributes $\mathbf{a_2}$ and $\mathbf{a_3}$?

Find the attribute concepts having $\mathbf{a_2}$ and $\mathbf{a_3}$ as intent and follow the lines down to the concept where they *meet*.

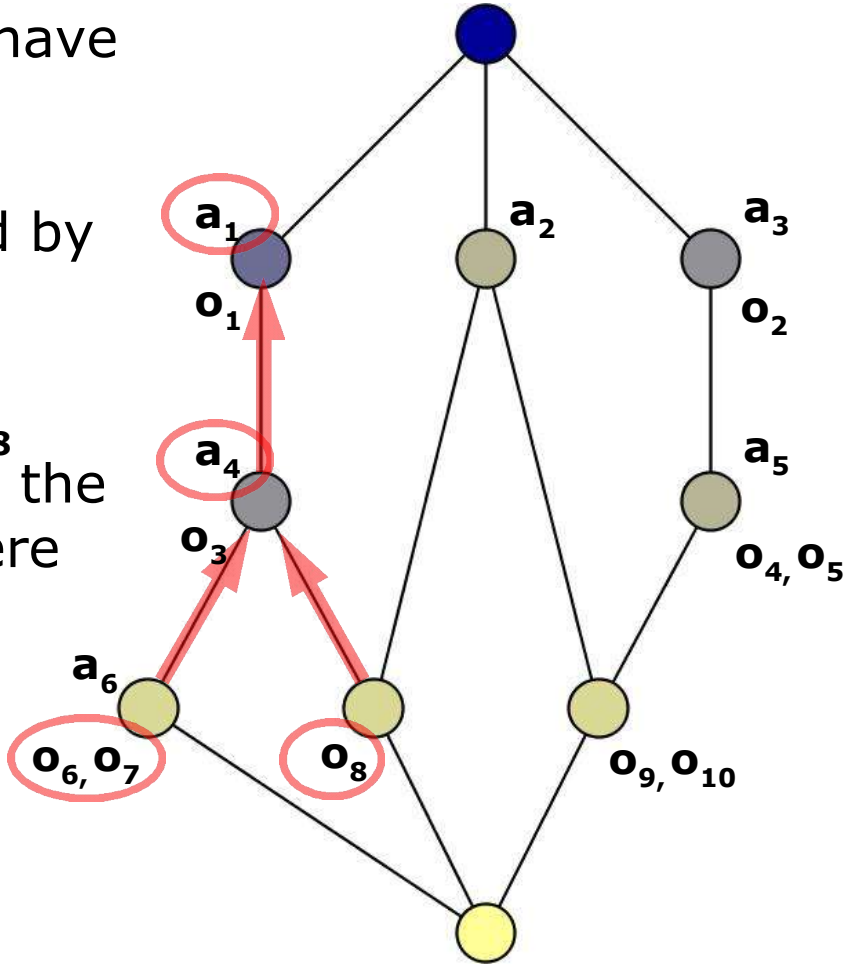


The *Concept Lattice:* Disjunction of concepts VII

Looking for attributes that have certain objects in common:

Which attributes are shared by objects $\mathbf{o_8}$ and $\mathbf{o_6}$?

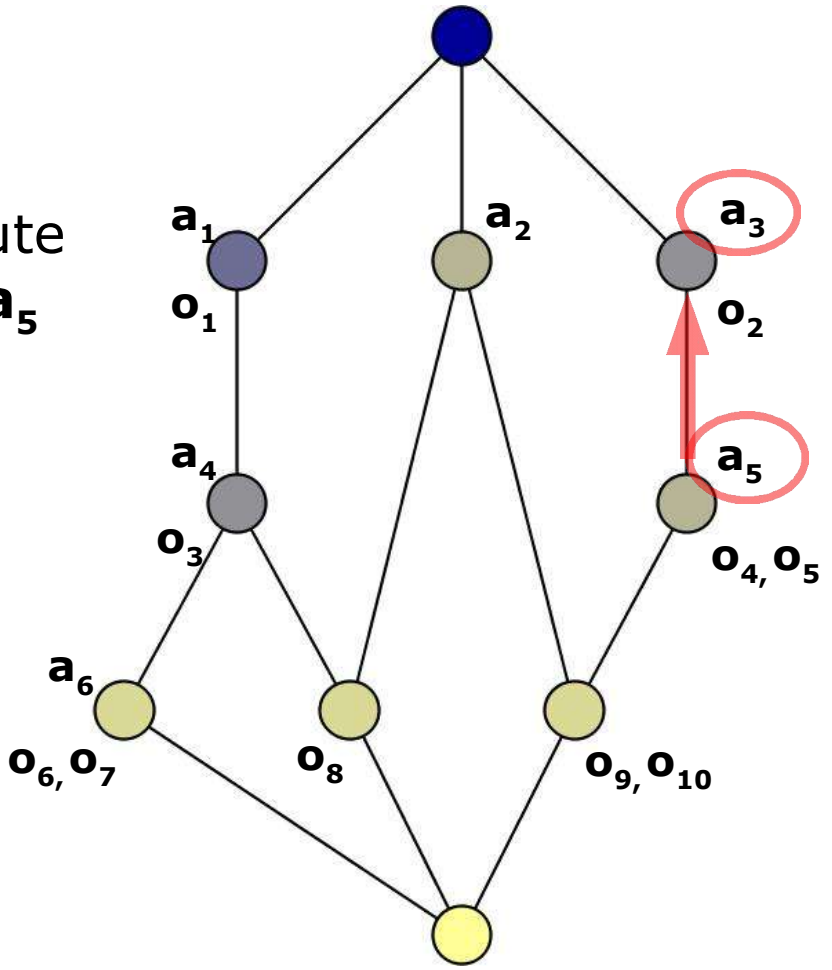
Find the concepts having $\mathbf{o_8}$ and $\mathbf{o_6}$ as extent and follow the lines up to the concept where they *join*.



The *Concept Lattice:* Implications VIII

Implications could also be identified:

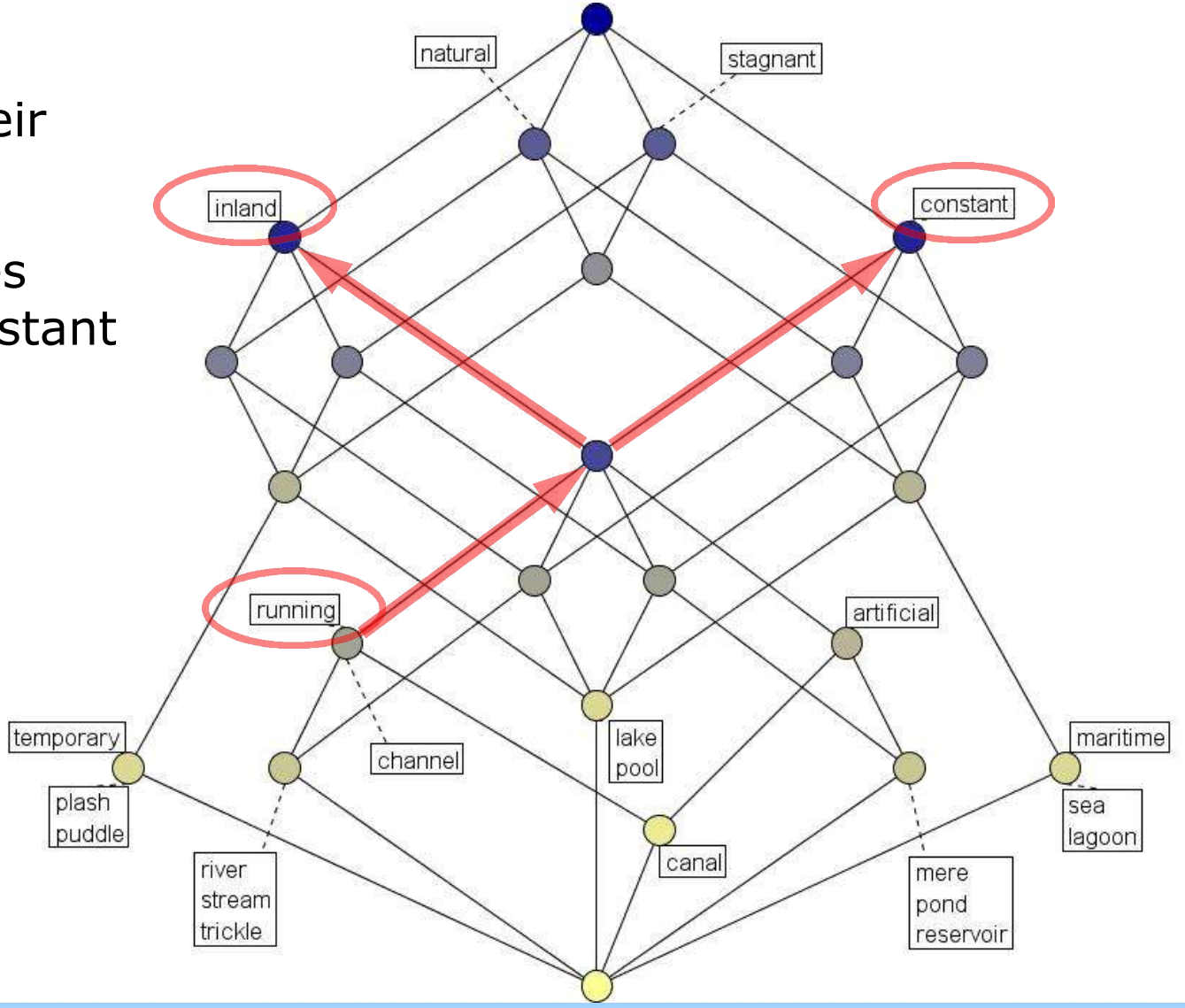
Every object that has attribute $\mathbf{a_5}$ also has attribute $\mathbf{a_3}$. So $\mathbf{a_5}$ implies $\mathbf{a_3}$.



The *Concept Lattice:* Implications IX

Waters and their attributes:

Running implies inland and constant

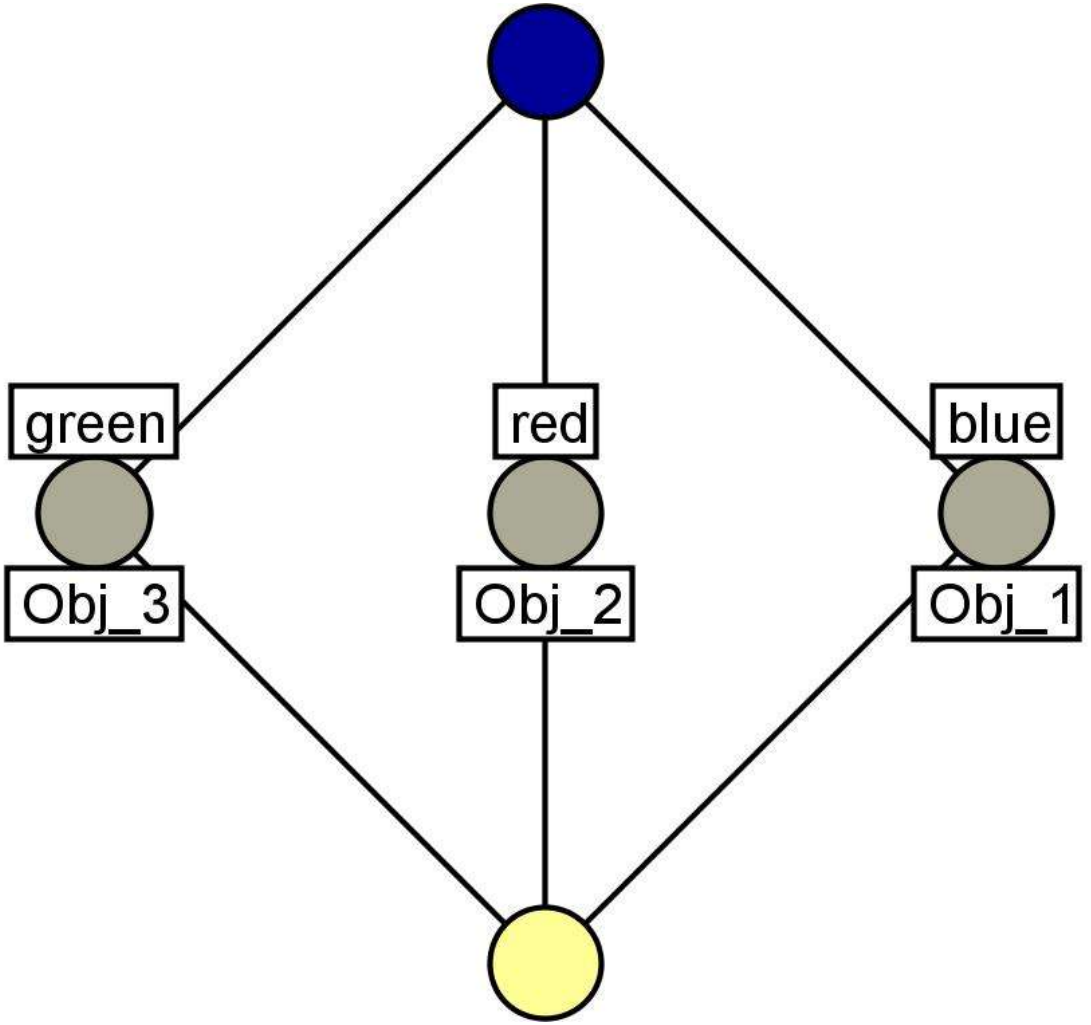


Conceptual Scaling I

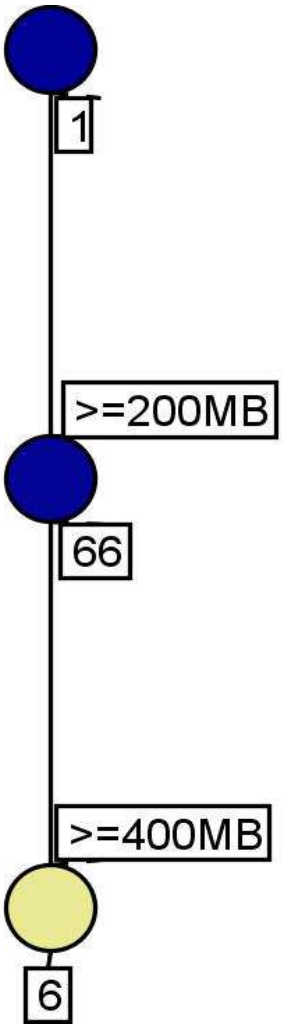
- A diagram based on a subset of attributes of a *formal context* is called a *conceptual scale*.
- The process of creating single-valued contexts from a many-valued data set is called *conceptual scaling.*
- *Conceptual scaling* could be standardized – but relies mostly on the human interpretation.
- The diagrams layout also influences the interpretation, different layouts might emphasize different aspects.
- Typically there is not a single best layout. It often depends on purpose and reader.
- There are some drawing "rules" to keep the layout neutral.
- Drawing a nice lattice automatically is an unsolved problem and still an area of research.

- Next: some standard scales ...

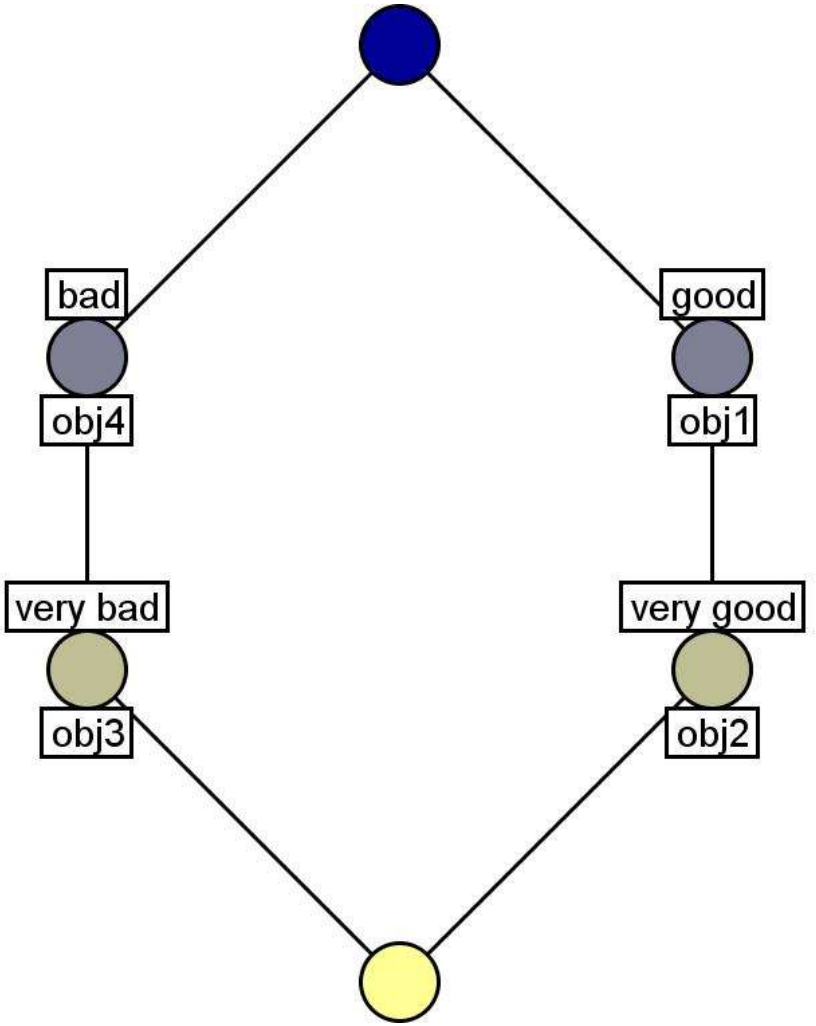
Conceptual Scaling: Nominal Scale II



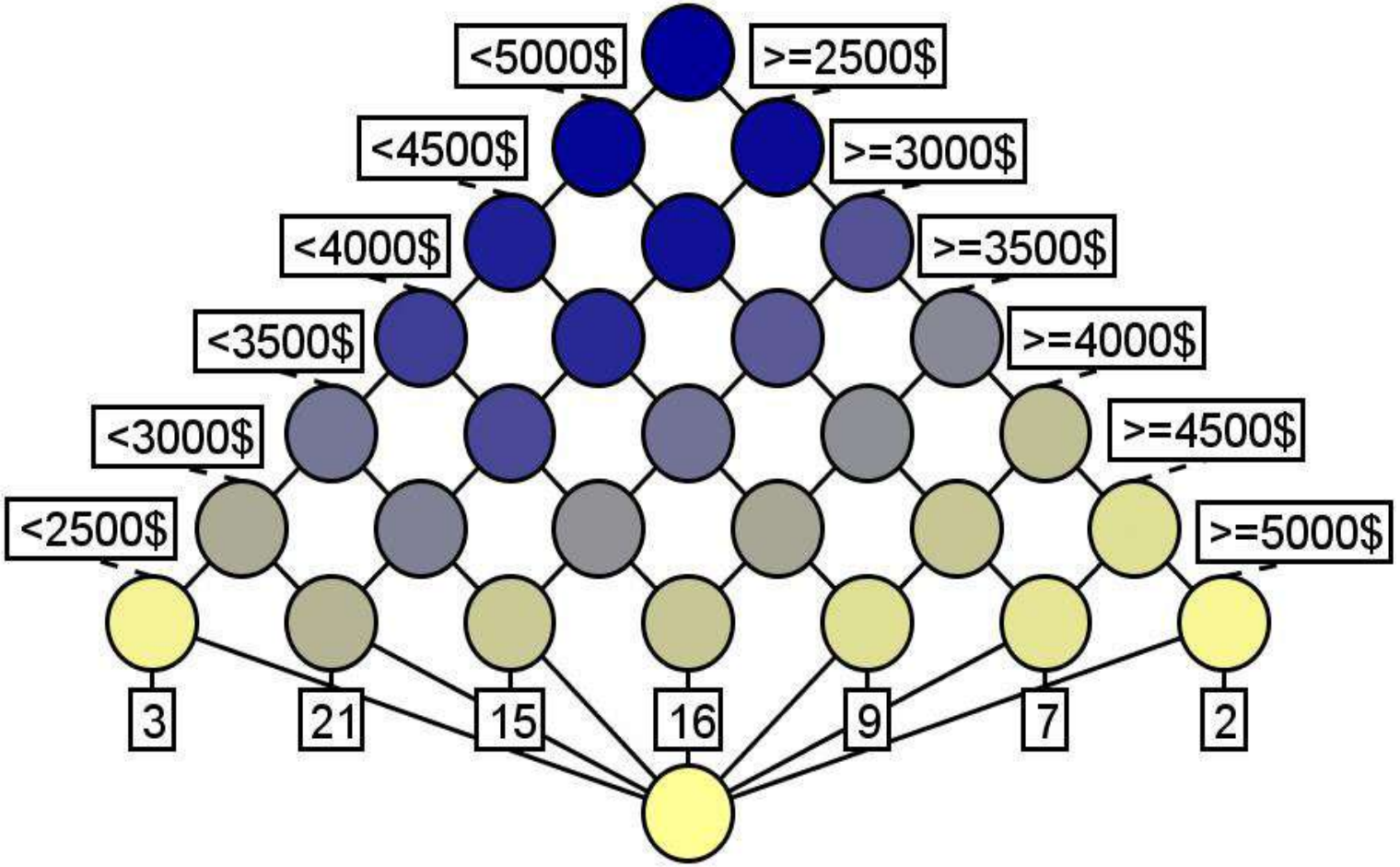
Conceptual Scaling: Ordinal Scale III



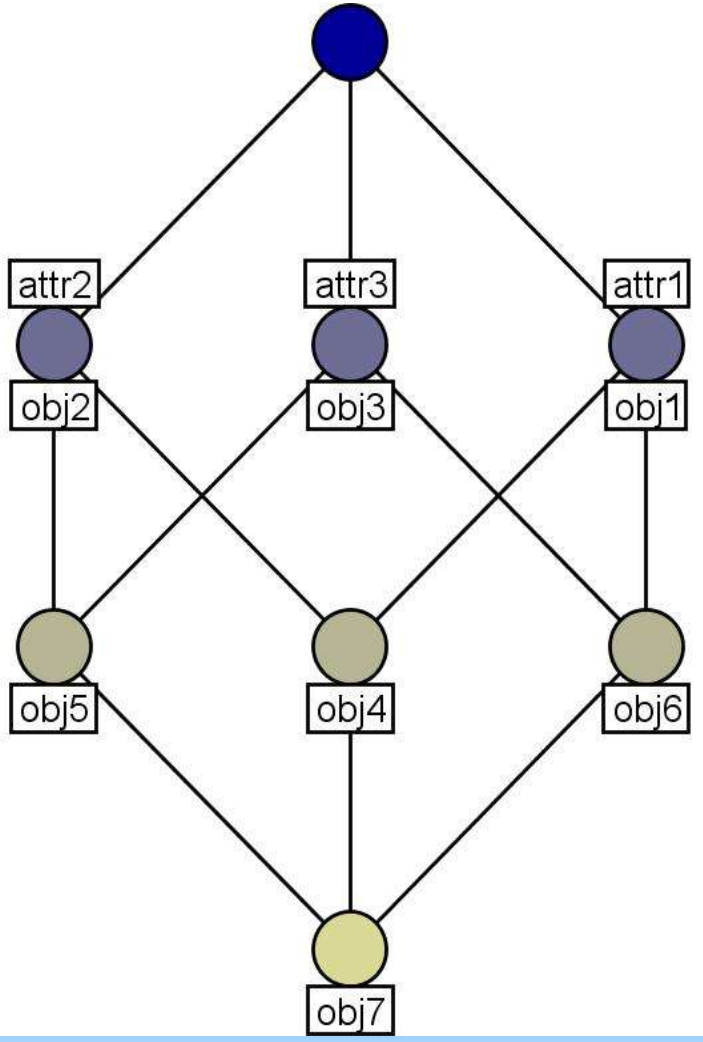
Conceptual Scaling: Biordinal Scale IV



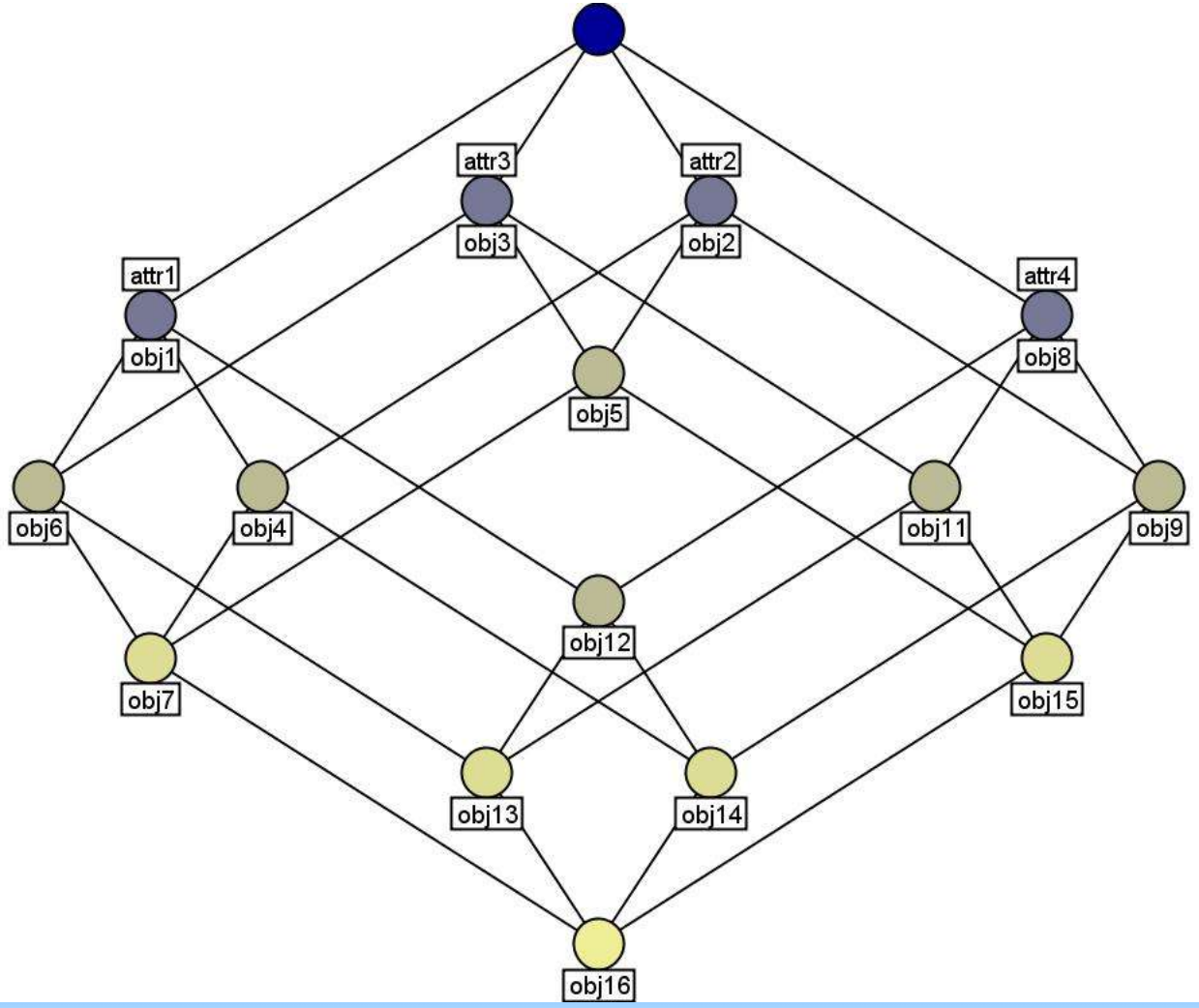
Conceptual Scaling: Interordinal Scale V



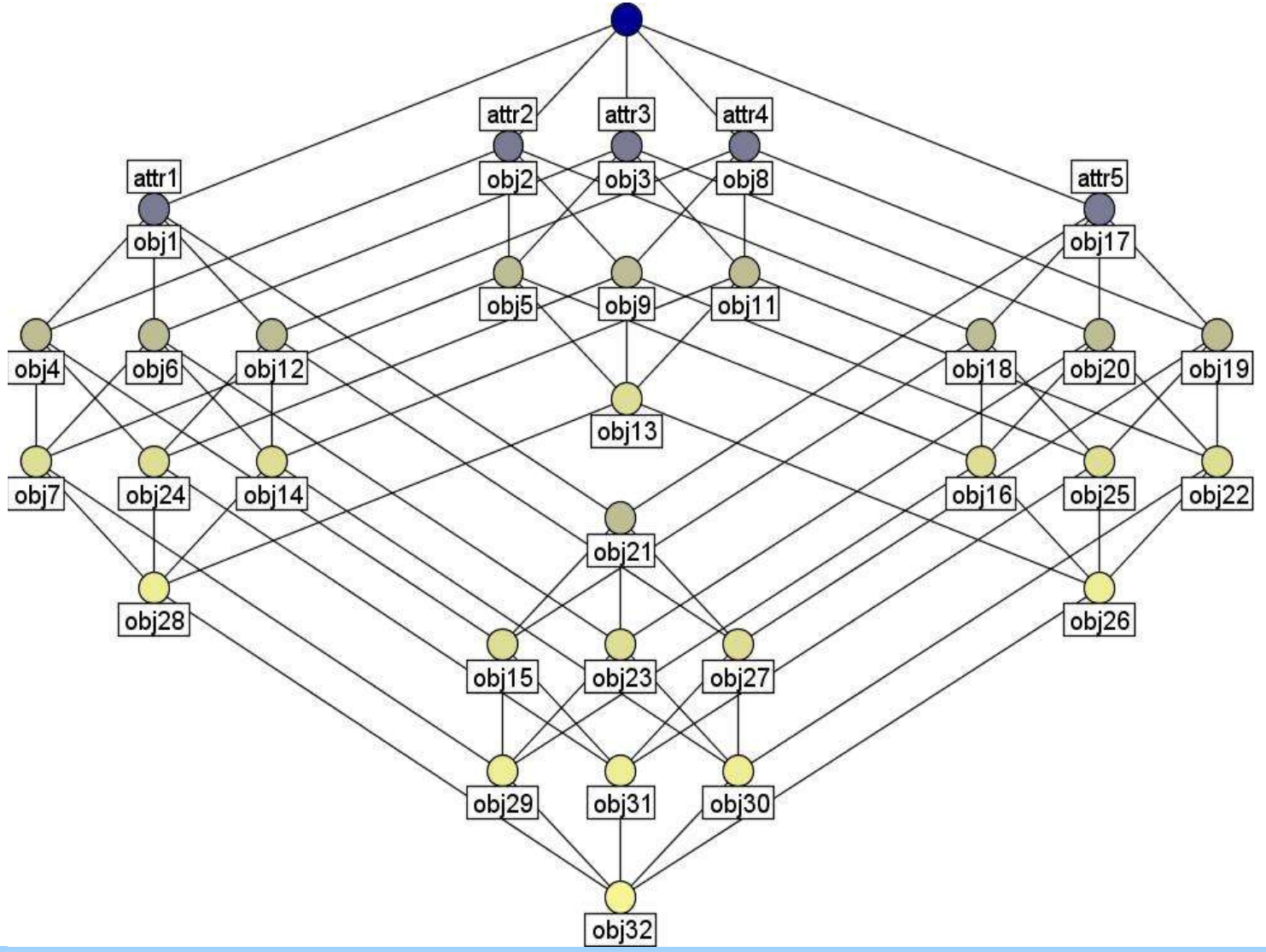
Conceptual Scaling: Boolean Scale VI



Conceptual Scaling: Boolean Scale VII



Conceptual Scaling: Boolean Scale VIII

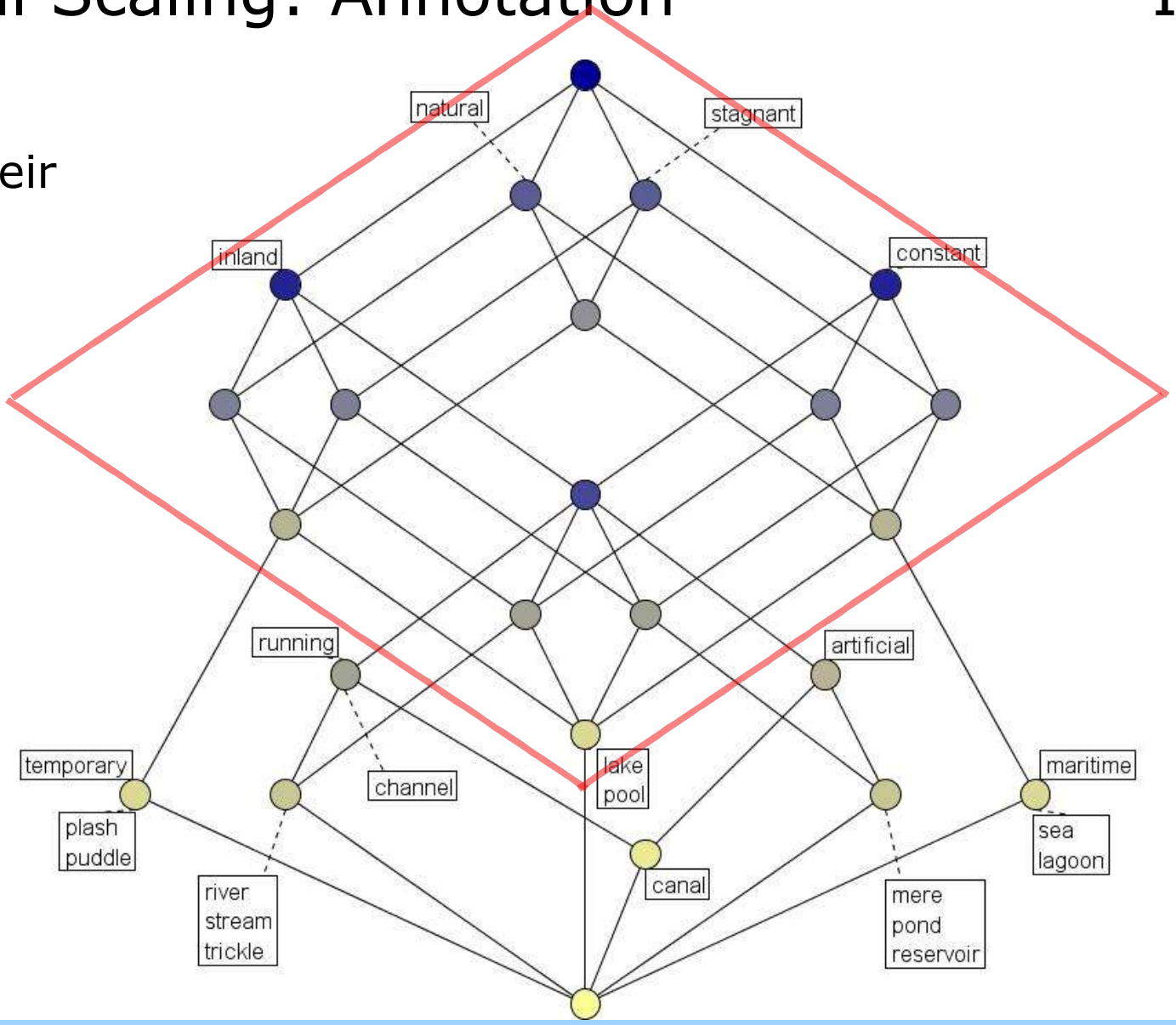


Annotation:

The scales shown represent typical data structures
-> you will recognize them or parts of them in your data

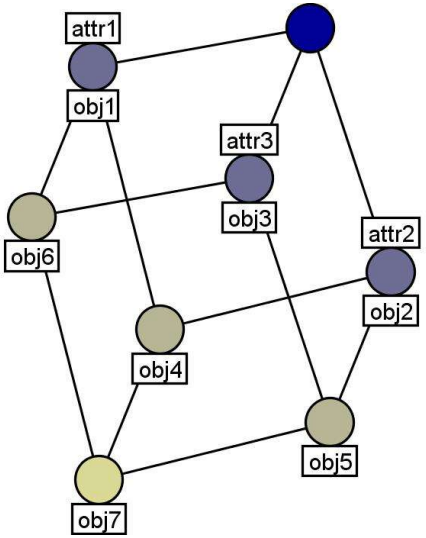
Conceptual Scaling: Annotation IX

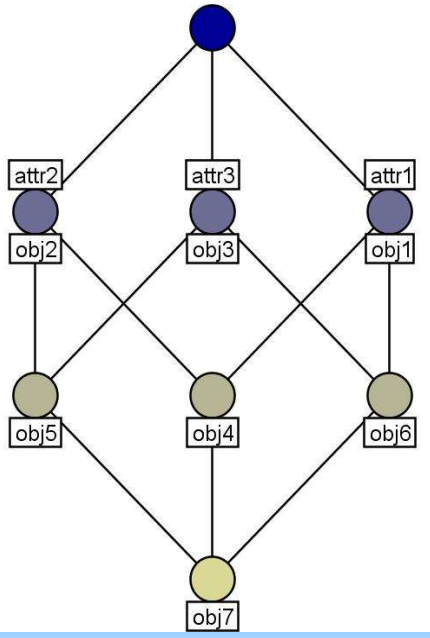
Waters and their attributes:



Lattice Drawing: Approaches I

- Avoid crossing of lines.
- Try to draw parallel lines.
- Identify known structures: cubes, rectangles ...
- Layer the nodes: draw nodes on the same layer, if their concept's extents have the same size.
- Try to draw steep lines, avoid flat ones.

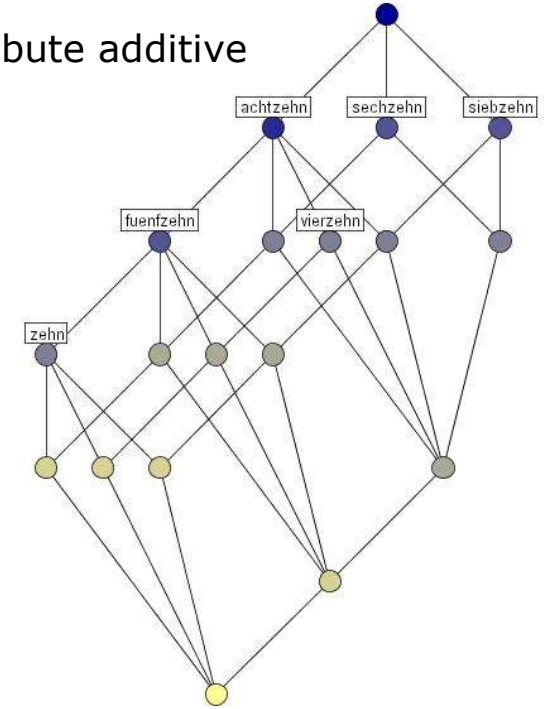




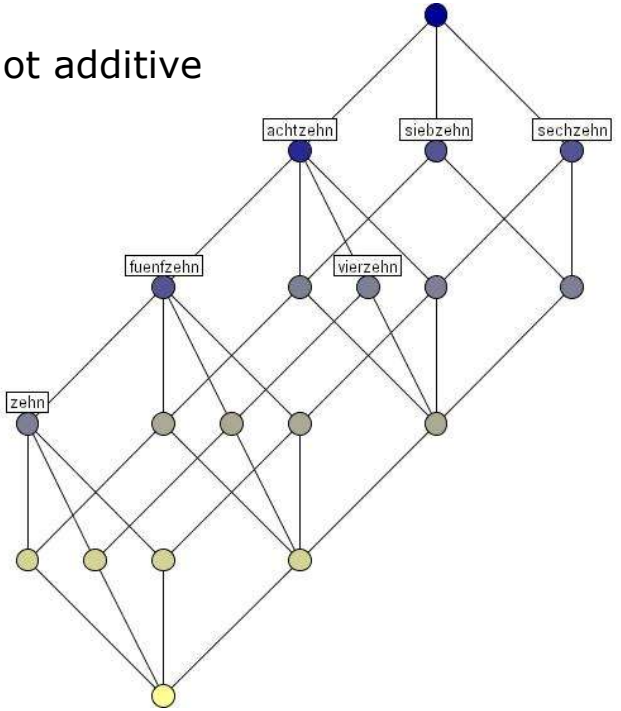
Lattice Drawing:Additive Line Diagrams II

- One approach is to assign vectors to the attributes.
- Each node's position is determined by the sum of all vectors of attributes in their concept's intent.

Attribute additive



Not additive



Summary

- Exploring the data: „playing“ with the data
- Regarding everything embedded in its conceptual surrounding
- The diagrams are stimulating discussions
- Identifying knowledge gaps and mistakes in the database
- FCA is creating a picture of people‘ s imagination
- ... could support data guided theory building

References / Sources:

Literature:

- B. Ganter/R. Wille: Formal Concept Analysis, Mathematical Foundations
- R. Wille: Introduction to Formal Concept Analysis

WWW:

- http://www.kvocentral.org
- This presentation could be downloaded via the conference homepage or at http://www.wormuth.info
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Thank you !